The Language of Mathematics

Frank Tapson

Mathematics generally has the reputation of having a precision that no other subject has, and no doubt at higher levels that is true. However, when we look closely at some of the words and phrases used in mathematics at school level, we might wonder whether that reputation is entirely justified, and I should like to draw attention to some of the words that can give rise to difficulties for various reasons. As teachers, we need to be aware of, and sensitive to, the way we need and use language to convey our meaning. Possibly we are not always as clear as we think.

Mathematicians might like to think that their particular language is a function by virtue of the fact that there is a one-to-one mapping between the words they use and the meanings of those of those words. A brief reflection will dispel that notion. Think of *square, cube, base, direct, inverse, tangent,* and a word such as *conjugate* has eleven different uses, though several of them are applicable only in mathematics beyond school level.

Starting with the basics, as long ago as in the time of Euclid (c.300 BC) it was realised that you have to start somewhere when you are defining terms. Words like *point, line, plane* and *equally likely* are impossible to define without getting into a circular argument. Most writers accept this by classing them as 'basic elements', 'common notions', 'undefined primitive concepts' and similar phrases or, more formally by declaring *axioms* or *posltulates* at the outset, which do not require to be proved.

There are various ways in which confusion can arise. One is where we have different words for what seem to be similar things. Another is where two or more words are needed to differentiate between things which are different in themselves, but which are a part of the same general topic. Yet another is where one word has different uses.

A pupil is given a small test. On a piece of paper are listed the words *quadrilateral, parallelogram, rhombus, rectangle, square.* The requirement is to draw a diagram that illustrates each of the words. There is one mark for each correct drawing. Alongside each word the pupil draws a square. What marks do you award? It is not difficulat to justify full marks being given, but is that really what we had in mind? See Appendix I for an illustration of the difficulties.

Some pairs of words which many pupils certainly have difficulty in distinguishing between are:

capacity & volume	complementary & supplementary	congruent & similar
bar chart & histogram	necessary & sufficient	probability & odds
possible & probable	sector & segment	recursion & iteration
discrete & continuous	interpolation & extrapolation	explicit & implicit
row & column	dependent & independent	sequence & series
converge & diverge	deduction & induction	convex & concave

and there is always that famous trio - mean, median, mode.

This is nothing to do with any ambiguity in definition, but because they refer to related concepts or objects which are connected in some way, and confusion arises over which word applies to which thing.

There are also words which we use in two different sense, usually leaving the context to make the distinction. For example, when we say *circle* do we mean the shape given by the line around it, or the space contained within that line, and that applies to every shape in both 2-D and 3-D. We rely upon the context to inform us. *Radius* is another example. In that case, we might mean the line itself (as in 'draw a radius') or the length of that line (as in 'find the radius of'). The latter of course should say 'find the length of the radius of' but often doesn't.

The widespread teaching of probability in comparatively recent times has given rise to the overloading of the word *event.* The dictionaries give two principal meanings for *event* as either a happening (e.g. throwing a die), or the result of that happening (e.g. getting a six). Unfortunately many writers seem to use the word in both senses and while, usually, it is reasonably clear which is which, it seems a great pity that the same word should be doing double duty within the same context. If *event* were restricted in use to its first meaning (= a happening), then the second meaning (= a result) would be dealt with by using only the word *outcome.*

Why is it that (traditionally) 2-dimensional shapes have *sides* and 3-dimensional shapes have *edges*? It is a usage I have never been happy with. In the *Longman's Mathematics Handbook* we read: '**edge** . . . Often used . . . unwisely for the side of a polygon', but the writer does not say why it is unwise. There are conflicts. Pick up a sheet of A4 paper. What shape is it? How many *sides* does that shape have? How many *sides* does the sheet of paper have? How many *edges* does the sheet of paper have? Then write a description of how to make a Mobius strip and go on to write about its properties in relation to the original strip of paper from which it was made - does there appear to be any confusion? Making a net produces a 2-dimensional shape having *sides* which can be folded up to make a 3-dimensional model having *edges*, so 2 *sides* = 1 *edge.* But what about all those extra *edges* which the model has (arising from the the fold-lines) and which were not present in the net? And our model now has *sides* - we might wish to call them *faces* but many pupils see them as *sides* in the real world. At least if we work with topological graphs we have the consistent *faces, vertices* and *edges.* The use by some writers of *regions, nodes* and *arcs* may help the imagery (in relating to networks), but they always have to explain the equivalence of those words when relating 3-dimensional models, Schlegel diagrams and Euler's law.

Do you ever use the word *oblong*? Though it appears to be rarely, if ever, used in mathematics, over many years I have never yet met a pupil who did not know what it meant when asked, but who nevertheless referred to it as a *rectangle* in the classroom. Closer questioning reveals that, to the majority, these two words are seen as being identical in meaning. The more general nature of a *rectangle* is rarely known. Given a sheet of drawings of various shapes and asked to put a tick in each *rectangle* there are not many pupils who will put a tick in the *square.* I wonder too about the writer who gave the instruction, 'Take a rectangle of paper which longer than it is wide . . .'. Why could he not use the word *oblong*? It is a matter of record that *oblong* has been in the English language for as long as *rectangle.* Let us bring *oblong* back into use and save *rectangle* for when we really do mean either *oblong* or *square.*

It is interesting to look at changes in language also. These do not usually lead to any confusion, but do show that we are not dealing with a standard (static) language, but that it is developing all the time. First a change that is now complete. The linguistically correct name for a polygon having nine edges is an *enneagon* (Greek prefix and stem), but who uses that nowadays? Universal usage has replaced it with *nonagon* (Latin prefix and Greek stem). Does it matter? Not so many years ago it was common to see the latter word used in books, but with a mention that the former word was more correct, but it has been a long time since I have seen a note to that effect.

A change that is currently taking place is in the description of numbers which make shapes. What always used to be known as *triangular numbers* are now more often referred to as *triangle numbers*. This seems to make sense. So, following on from that, should we not use *polygon numbers* as the general term and *pentagon-, hexagon-* etc. *numbers* for the specific types?

Plurals provide a good example of changes that can be observed. It is not difficult to find two different forms in current use. The most recent case I have noted is *formulae* and *formulas*. The English way of forming plurals is slowly gaining ground but how far can or will it go? I find I can accept (and use without faltering) *polyhedrons, formulas, trapeziums, rhombuses, apexes, hyperbolas, parabolas,* but draw back from *matrixes, radiuses, vertexes, locuses,* and am ambivalent about the plurals of *maximum, minimum, helix* - depending upon my audience. I suppose that almost sixty years of usage has ingrained certain practices in me and I cannot shrug them off that easily. And have you noticed that whereas books have *indexes,* mathematics has *indices*?

The battle over *data* seems to be almost over. It is now increasingly treated as a singular collective noun, giving us *the data is* rather than the awkward sounding *the data are*. The latter still has a use when it is desired to emphasise the fact that the group is actually made up of several separate pieces of information. As for *dice*, I have yet to meet a pupil who readily uses the singular *die* and have come to accept *1 dice* and *2 dice* (after all we have a good precedent in *sheep*), but draw the line at *2 dices*!

Another problem with the language of mathematics is that it sometimes seems to be unnecessarily complicated when compared with ordinary everday language. For instance, *pyramid* means one thing only to most people, and is equated with those found in Egypt. However, for sound mathematical reasons we find it necessary to describe such things as *right, square-based pyramids* though, at school level, we rarely mean any other sort. *Cylinders* and *cones* are treated similarly. Necessary yes, but we should remain aware of the strain this can impose upon the reading and understanding of many. At least *cubes* and *spheres* are unambiguous.

To finish off, a few for you think about.

- How many uses or meanings can you find for the words *sum, order, net, base, vertex* and *square?*
- Why does every 2-dimensional shape have a *perimeter* except for the circle and ellipse which have a *circumference,* and would it be of benefit to standardise?
- Can you use (or explain) the words: *length, breadth, width, height, depth, thickness* without any ambiguity?
- What do we mean by each of these: *number, digit, numeral* and *figure*?
- What exactly is the difference between *curve, line, straight line* and *line segment?*
- Just what do pupils understand by *invariably?*

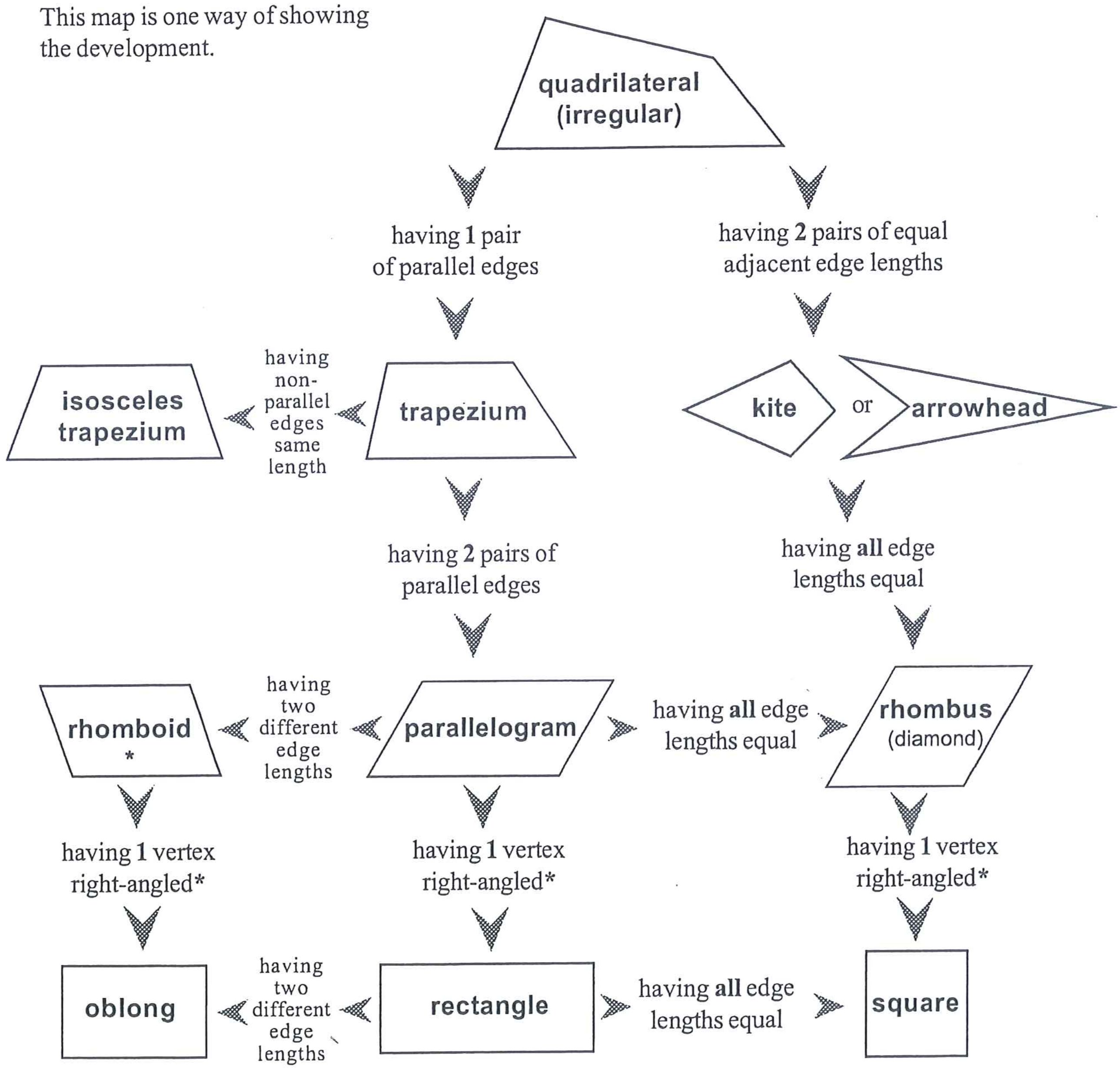
Frank Tapson was formerly Head of Mathematics at a Comprehensive School and now works at the Centre for Innovation in Mathematics Teaching, based at the University of Exeter. He is the author of the *Oxford Mathematics Study Dictionary.*
Contact: CIMT, School of Education, University of Exeter, Exeter, EX1 2LU
or by Email to: F.Tapson@ex.ac.uk

The Language of Mathematics ~ Appendix I

The classification of quadrilaterals is a mathematical topic that is often written about. Usually the writers show different ways of doing it, illustrating their system by using either a 'tree' approach or a Venn diagram. Most seem to conclude that any system is 'unsatisfactory' in some way or other and, often, produce a new name for a shape in order to regularise the system. This might give some idea of why pupils can have difficulties at times in trying to understand what we are driving at. Looking at the definitions in nearly all major dictionaries and we see that:

a square is a rhombus is a parallelogram is a trapezium is a quadrilateral (is a polygon).

This map is one way of showing the development.



rhomboid is an old name. Unfortunately it is also used, in crystallography, as an alternative name for a rhombohedron.

See how the insertion of the word 'only' in some places, would change matters greatly.

A good classroom exercise is to change the above into a Venn diagram.

The expression 'having 1 vertex right-angled' is minimal - it follows that all of them must be.